Calculus Warm Up #4-1

The radius of a sphere is increasing at a constant rate of 0.04 cm/sec. At the time when r = 10 cm, what is the rate of increase of the volume of the sphere?

10.5 Alternating Series

The Alternating Series Test for Convergence

Absolute Convergence

Conditional Convergence

Alternating Series

For $a_n > 0$, the terms alternate signs.

Odd terms negative:

$$\sum_{n=1}^{\infty} (-1)^n a_n = -a_1 + a_2 - a_3 + a_4 - \cdots$$

Even terms negative:

$$\sum_{n=1}^{\infty} (-1)^{n-1} a_n = a_1 - a_2 + a_3 - a_4 + \cdots$$

Alternating Series Test

The series will converge if these 2 conditions are met:

1) $\lim\limits_{n\to\infty} a_n = 0$ 2) $0 < a_{n+1} \le a_n$

Example:

$$\sum_{n=1}^{\infty} \frac{n}{(-2)^{n-1}}$$

$$\rightarrow \lim_{n\to\infty} \frac{n}{2^{n-1}} = \frac{\infty}{\infty}$$

$$= \lim_{n\to\infty} \frac{1}{(\ln 2) 2^{n-1}} = 0 \checkmark$$

a_1	a_2	a_3	a_4	a_5
$\frac{1}{1}$	$\frac{2}{2}$	$\frac{3}{4}$	$\frac{4}{8}$	$\frac{5}{16}$

$$a_{n+1} \le a_n$$

$$\frac{n+1}{2^n} \le \frac{n}{2^{n-1}} \quad \checkmark$$

Series Converges by Alt. Series test

Ex. $\sum_{n=1}^{\infty} \frac{(-1)^n n}{\ln(2n)}$

let $f(x) = \frac{x}{\ln(2x)}$

$\lim_{x\to\infty} \frac{x}{\ln(2x)} = \frac{\infty}{\infty}$

$\lim_{x\to\infty} \frac{1}{\frac{1}{2x}\cdot 2}$

$\lim_{x\to\infty} x = \infty$

$\therefore$ Alt Series Test does not apply.

Series diverges by n^{th} term test.

Ex. $\sum_{n=1}^{\infty} (-1)^{n+1}\left(\frac{3n+2}{4n^2-3}\right)$

let $f(x) = \frac{3x+2}{4x^2-3} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$

$\lim_{x\to\infty} \frac{\frac{3}{x}+\frac{2}{x^2}}{4-\frac{3}{x^2}} = \frac{0}{4}$

$f'(x) = \frac{(4x^2-3)(3)-(3x+2)(8x)}{(4x^2-3)^2}$

$= \frac{12x^2-9-24x^2-16x}{(4x^2-3)^2}$

$f'(x) = \frac{-(12x^2+16x+9)}{(4x^2-3)^2}$

f is decreasing b/c $f'(x) < 0$

$\therefore a_{n+1} \leq a_n$

Series converges by Alt. Series Test

HW: p. 598, # 1 - 21 odd, and

check answers for purple & pink FR

This week: more of Chapter 10

more FR practice

group quiz Friday over 10.1 - 10.4

Classwork: Continue with

2002 FR # 1 - 3 (Purple)

A: intersection
$x \approx 1.488$
stored.

1a) $A = \int_0^A \left[4 - 2x - \left(\frac{x^3}{1+x^2}\right)\right] dx$

$A \approx 3.215$

b) $R = 4 - 2x$

$r = \frac{x^3}{1+x^2}$

$V = \pi \int_0^A \left[(4-2x)^2 - \left(\frac{x^3}{1+x^2}\right)^2\right] dx$

$V \approx 31.885$

c) $V = \int_0^A \left(4 - 2x - \left(\frac{x^3}{1+x^2}\right)\right)^2 dx \approx 8.997$

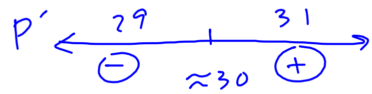
2a) increasing if $P'(9) = +$

$P'(9) \approx -0.646$, so amount of P is decreasing at $t = 9$ days

b) min where $P'(t) = 0$ & $P'(t)$ goes from $-$ to $+$

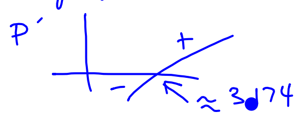
$0 = 1 - 3e^{-0.2\sqrt{t}}$

$t \approx 30.174$ days



confirms min at $t \approx 30.1$ days

* could also graph:



c) Safe if $P(\approx 30.1) \leq 40$

$P(\approx 30.1) = 50 + \int_0^{\approx 30.1} P'(t)\,dt$

≈ 35.104

Since $35.104 < 40$, yes the lake is safe when P is at its minimum.

2d) tangent line: $y - 50 = -2(t - 0)$

$y = -2t + 50$

Safe when $y \leq 40$

so: $-2t + 50 \leq 40$

$t \geq 5$

linear model predicts the lake becomes safe when $t \geq 5$ days.

3a) graph

b) particle moves left when $v(t) < 0$

on $(\pi, 2\pi)$; $(3\pi, 4\pi)$; $(5\pi, 16]$

c) Total distance : Ab. Value of Area under curve $[0,4]$

$$\int_0^{\pi} v(t)\,dt - \int_{\pi}^{4} v(t)\,dt \approx 10.542$$

d) Particle never returns to the origin b/c

$v(t)$ is more $+$ than $-$, so $\int v(t)\,dt > 0$

Pink FR 2008 #3

a) $\frac{dh}{dt} \approx 0.039$ cm/min

b) $t = 25$ minutes

c) $V = 60{,}000 + \int_0^{25} (2000 - 400\sqrt{t})\,dt$ cm^3